



**Minnesota Graduation-Required  
Assessments for Diploma  
(GRAD)**

**Test Specifications for Mathematics**

**August 16, 2010**

# MINNESOTA DEPARTMENT OF EDUCATION

## GRAD Test Specifications for Mathematics

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For a copy in an alternate format, contact

Division of  
Research and Assessment  
1500 Highway 36 West  
Roseville, MN 55113-4266  
651-582-8200  
mde.testing@state.mn.us

© Minnesota Department of Education  
1500 Highway 36 West  
Roseville, MN 55113-4266  
Phone 651-582-8200 • Fax 651-582-8874

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## **THE GRADUATION-REQUIRED ASSESSMENTS FOR DIPLOMA**

### *Introduction*

The 1997 legislature enacted Minnesota Statute § 121.113 (1997), Statewide Testing and Reporting System, which established annual testing of all students in grades eight (reading and mathematics) and ten (written composition). This legislation established the Basic Skills Tests, which all students were required to pass in order to graduate from a Minnesota public high school.

The Minnesota legislature later enacted the Omnibus K–12 and Early Childhood Act of 2005 that replaced the Basic Skills Tests given in high school. This statute requires students enrolled in grade 8 before the 2005–2006 school year to pass the Basic Skills Tests. The statute requires that students enrolled in grade 8 in the 2005–2006 school year or later must obtain an achievement level equivalent to or greater than proficient on the Minnesota Comprehensive Assessments-Series II (MCA-II) in reading and math or pass the Graduation-Required Assessments for Diploma (GRAD) in reading and math. Students enrolled in grade 8 in the 2005–2006 school year or later must pass the GRAD in writing.

In the 2007 legislative session, the statute was revised to include options for retest opportunities as well as students taking other assessments to meet the graduation-testing requirement.

In the 2009 legislative session, the statute changed the mathematics requirements for students in the classes of 2010–2014. Students in these five classes are not required to obtain a proficient score on the Mathematics MCA-II or required to pass the Mathematics GRAD in order to be eligible to graduate from high school. Students must complete all coursework and credits required for graduation, participate in district-prescribed academic remediation in mathematics and participate in at least two retests of the Mathematics GRAD or pass the Mathematics GRAD, whichever comes first.<sup>1</sup>

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<sup>1</sup> Information contained within this document is reflective of the current GRAD legislation as of the date of this document's publication.

## **GRAD** **Test Specifications**

### ***Purpose of the Graduation-Required Assessments for Diploma***

The GRAD measures the writing, reading and mathematics proficiency of high school students. By requiring high school graduates to reach a specified level on each of these assessments, Minnesota is making sure its students are on track to have the essential skills and knowledge necessary for graduation in the 21<sup>st</sup> century.

### ***Purpose of the Test Specifications***

All tests, from off-the-shelf, norm-referenced tests (NRT) to customized, standards-based tests like those given in Minnesota, have test specifications. The primary purpose of a set of test specifications is to help test developers build a test that stays consistent over time. Test specifications indicate which strands, substrands, standards and benchmarks will be assessed on the test and in what proportions. In addition, test specifications provide the number of items and the type of items to be included. Test specifications also clarify, define and/or limit how test items will be written to any given strand, substrand, standard or benchmark.

Test specifications do not indicate **what** should be taught: the Academic Standards do. Test specifications do not indicate **how** children should be taught: the classroom teacher does.

The test specifications presented in this document were developed over the course of many days by Minnesota teachers, many of whom were recommended by various education organizations, school districts, and other stakeholder groups. The substantive parts of this document are true to their hard work. The Department thanks these people for their effort and continued involvement.

The [Minnesota Academic Standards](http://education.state.mn.us) can be obtained from the Department of Education Website (<http://education.state.mn.us>. > Academic Excellence > Academic Standards).

### ***Administration of the Graduation-Required Assessments for Diploma***

The first administration of the Mathematics GRAD is embedded within the Mathematics MCA-II grade 11 assessment. If a student does not obtain an achievement level equivalent to or greater than proficient on the MCA-II in Mathematics or a passing score on the Mathematics GRAD, then the student is eligible to retake the Mathematics GRAD.

The benchmarks included on the Mathematics GRAD originate from the 2003 Minnesota K–12 Academic Standards in Mathematics. A committee of Minnesota teachers identified the essential

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skills and knowledge, based on the Minnesota K–12 Academic Standards, necessary for graduation in the 21<sup>st</sup> century. Some of the benchmarks the teachers identified appear on both the GRAD and the MCA-II Grade 11 Mathematics assessment and are labeled “Common” in the GRAD test specifications. Additional benchmarks were identified from the Minnesota K–12 Academic Standards in Mathematics and are administered only on the GRAD and are labeled “GRAD” in the GRAD test specifications.

**GRAD Administrations**

<b>COMMON COMPONENT 25 ITEMS</b>	<b>GRAD-ONLY COMPONENT 15 ITEMS</b>	<b>TOTAL TEST = 40 ITEMS</b>
II. Number Sense III. Patterns, Functions and Algebra IV. Data, Statistics and Probability V. Spatial Sense, Geometry and Measurement	II.G Number Sense III.G Patterns, Functions and Algebra IV.G Data, Statistics and Probability VG. Spatial Sense, Geometry and Measurement	

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# MATHEMATICS GRAD

## *Overall Considerations in Mathematics*

As stated in the **Purpose of Test Specifications**, the test specifications indicate which strands, substrands, standards and/or benchmarks will be assessed. The following points specifically address the math test specifications of the GRAD.

Overall considerations are issues that should be addressed during the development of test items. Each of these issues should be considered for all of the items developed for the GRAD in mathematics.

1. Each item should be written to measure primarily one benchmark; however, other benchmarks may also be reflected in the item content. When benchmarks are combined for assessment, the individual specification indicates which benchmarks are combined.
2. Items should range in difficulty from easy to challenging.
3. Items should not disadvantage or disrespect the age, gender, race, ethnicity, language, religion, socioeconomic status, disability or geographic region of any segment of the population.
4. Each item should be written clearly and unambiguously to elicit the desired response.
5. A reference sheet of appropriate formulas and conversions is provided to students during testing.
6. Items should be written according to the MDE Guidelines for Test Construction.
7. Advisory Panels will review items as specified in the MDE Vendor Guide to Advisory Panels.
8. Items are reviewed for content characteristics, potential bias and other issues that may be of concern. Minnesota educators with experience and expertise in mathematics instruction at the high school level review the items in terms of content (item difficulty) and bias (gender, racial/ethnic, linguistic, religious, geographic, socioeconomic and issues related to individuals with disabilities).
9. After field-testing, Minnesota educators with appropriate experience and expertise review the test data for each item that has been field-tested. Each item will be reviewed in terms of its bias and psychometric data.

## Mathematics GRAD

### Mathematics Test Design

#### Mathematics Item Percentages by Strand and Substrand

These tables provide an overview of percentage of items in the tests by the strands (Roman numerals) assessed by the tests.

<b>Strand II:</b> Number Sense <b>Strand III:</b> Patterns, Functions and Algebra <b>Strand IV:</b> Data, Statistics and Probability <b>Strand V:</b> Spatial Sense, Geometry and Measurement
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#### 40 Items

	Strand			
	II	III	IV	V
Common		36–52%	20–36%	20–36%
GRAD Only	33–53%	20–40%	20–33%	0–13%
TOTAL TEST	13–20%	30–48%	20–35%	15–28%

#### Mathematics Item Distribution

This table provides the total number of items and points for each strand on the GRAD. All items on the GRAD are multiple-choice questions.

#### GRAD

	Items				TOTAL ITEMS	TOTAL POINTS
	Strand II	Strand III	Strand IV	Strand V		
	MC	MC	MC	MC		
Common		9–13	5–9	5–9	25 MC	25 PTS
GRAD Only	5–8	3–6	3–5	1–2	15 MC	15 PTS
TOTAL TEST	5–8	12–19	8–14	6–11	40 MC	40 PTS

MC: multiple-choice questions

PTS: points assigned

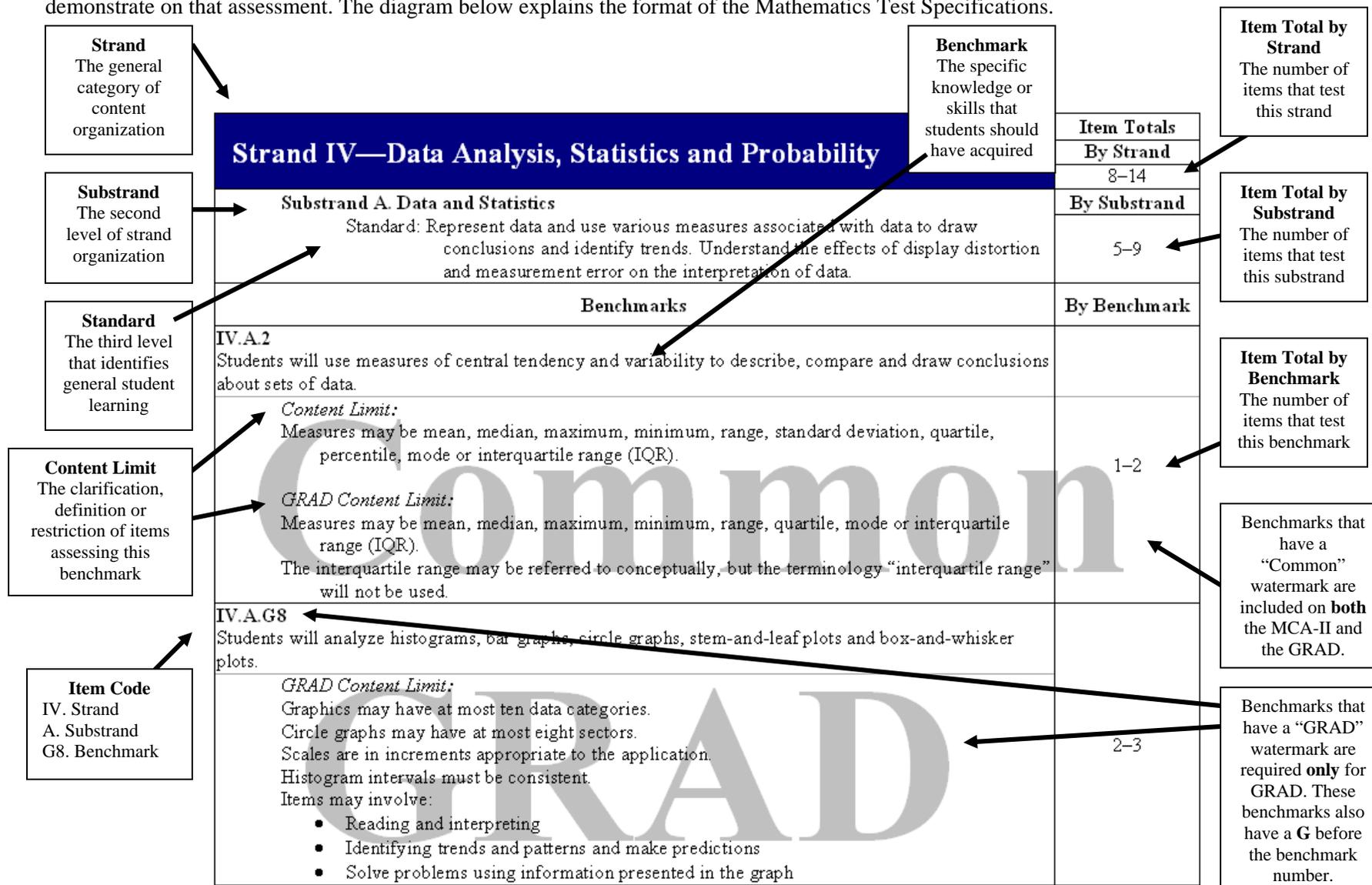
#### Calculator Usage

Calculators may be used for all questions on the Mathematics GRAD. However, a calculator is not required to answer questions.

## Mathematics GRAD

### A Guide to the Mathematics Tables for GRAD

The GRAD Test Specifications provide information about the benchmarks assessed on the GRAD and the skills and knowledge that students must demonstrate on that assessment. The diagram below explains the format of the Mathematics Test Specifications.



## *Mathematics GRAD*

### **An Explanation of Terms on the Mathematics GRAD Tables**

**Strand:** This is the most general categorization of content in the Minnesota Academic Standards.

**Substrand:** This is a subcategory of a strand in the Minnesota Academic Standards. Mathematics has two to three substrands in each strand.

**Standard:** This statement explains the general goal of student learning within each substrand. One standard exists in each substrand.

**Benchmark:** Each standard is divided into several benchmarks. The benchmark identifies the specific knowledge or skills that students should have acquired.

**Item Code:** Test developers use this code to identify the strand, substrand and benchmark to which a test item is aligned. Benchmarks that are GRAD only have a **G** before the benchmark number.

**Content Limit:** These statements provide more specific clarifications, definitions or restrictions for the benchmark. Benchmarks that are included on the MCA-II as well have *GRAD Content Limit* listed below the MCA-II *Content Limit*. Items assessing the benchmarks that have a “Common” watermark must meet all of the benchmark’s content limits in order to be on the operational GRAD. If an item written to a common benchmark does not meet the GRAD Content Limits, then it does not count for a GRAD score.

#### **Item Totals**

**By Strand:** This number is the possible number of items that will be on the operational GRAD form from a specific strand.

**By Substrand:** This number is the total number of items measuring the substrand for the indicated standard that could be on the GRAD.

**By Benchmark:** The number of items on the operational GRAD is listed next to each benchmark.

Item Specifications

<b>Strand II—Number Sense</b>	<b>Item Totals</b>
	<b>By Strand</b>
	5–8
<b>Substrand B. Computation and Operation</b> Standard: Appropriately use calculators and other technologies to solve algebraic, geometric, probabilistic and statistical problems.	<b>By Substrand</b> 5–8
<b>Benchmarks</b>	<b>By Benchmark</b>
<p><b>II.B.G7</b> Students will apply the correct order of operations to simplify and evaluate numeric expressions. <i>GRAD Content Limit:</i> Items must use positive rational numbers. Items will use addition, subtraction, multiplication, division and grouping symbols only. Fraction denominators are limited to 2, 3, 4, 5, 6, 8, 10 and 12. Subtraction cannot be a mixed number minus a mixed number requiring regrouping (e.g., <math>3\frac{1}{4} - 1\frac{7}{8}</math> is not acceptable) Multiplication may be expressed as raised dot, x, or ( ) (e.g., <math>5 \bullet 6</math>, <math>5 \times 6</math>, <math>5(6)</math> ). Division may be expressed using division symbol or fraction bar (e.g., <math>6 \div 2</math> or <math>\frac{6}{2}</math> ). Division items must have a whole number divisor. For multiplication and division items, mixed numbers must be expressed as improper fractions. No nested grouping symbols are allowed (e.g., <math>3[292 + (100/2)]</math> is not allowed). Items may require the identification of the correct order of operations shown (calculation not required). Items may include exponents. Problems may include context.</p>	2–3
<p><b>II.B.G8</b> Students will use rational numbers in complex ways to solve multi-step real-world and mathematical problems. <i>GRAD Content Limit:</i> Rationals are limited to positive rationals. Non-integer rationals will be represented in decimal form. Items may require integer approximations of square roots of positive integers. Squares must be less than or equal to 150.</p>	0–1

<p><b>II.B.G9</b>  Students will use fractions, decimals and percents in multiple representations for estimation and computation to solve real-world and mathematical problems.  <i>GRAD Content Limit:</i>  Fraction denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25 and 100. Restrictions on denominators apply to problem <u>and</u> answer options. (e.g., <math>1/3 + 1/5</math> is not allowed; <math>1/15 - 1/3</math> is not allowed).  Items may include positive and negative fractions, decimals and percents.</p>	<p>1–2</p>
<p><b>II.B.G10</b>  Students will use proportional reasoning to solve real-world and mathematical problems.  <i>GRAD Content Limit:</i>  Items may involve:</p> <ul style="list-style-type: none"> <li>• Rates</li> <li>• Scale drawings and maps</li> <li>• Similar figures</li> <li>• Ratio</li> <li>• Unit pricing</li> <li>• Showing how changing one or more dimensions affects change in area.</li> </ul> <p>Shapes are limited to circles, parallelograms and triangles.  Items are limited to two-dimensional figures.  Pictures or diagrams may be used but are not required.  Similarity may be shown using similarity symbol (<math>\sim</math>) or using markings on figures.  Items may include context.</p>	<p>2–3</p>

<b>Strand III—Patterns, Functions and Algebra</b>		<b>Item Totals</b>
		<b>By Strand</b>
		12–19
<b>Substrand A. Patterns and Functions</b>		<b>By Substrand</b>
Standard: Represent and analyze real-world and mathematical problems using numeric, graphic and symbolic methods for a variety of functions.		5–7
<b>Benchmarks</b>		<b>By Benchmark</b>
<p><b>III.A.1</b> Students will know the numeric, graphic and symbolic properties of linear, step, absolute value and quadratic functions.</p> <p><i>Content Limit:</i> Items may include rates of change, intercepts, maxima and minima. Items may include intersection between two graphs. Step functions must model real-world situations. Step functions will not be represented symbolically.</p> <p><i>GRAD Content Limit:</i> Items may not include step or absolute value functions. Items may include rates of change and intercepts. Items that assess quadratics are limited to graphical properties. Increments of <math>x</math> and <math>y</math> axes must be integers.</p>		1–2
<p><b>III.A.2</b> Students will model exponential growth and decay.</p> <p><i>Content Limit:</i> Models may be numeric, graphic and symbolic. When calculation is required, exponents must be integers. Items may have real-world context (e.g., bacterial growth, half-life, compound interest).</p> <p><i>GRAD Content Limit:</i> Models may be numeric or graphic. Items may have real-world context (e.g., bacterial growth, half-life, compound interest).</p>		1–2

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<p><b>III.A.4</b>                  Students will apply basic concepts of linear, quadratic and exponential expressions or equations in real-world problems.  <i>Content Limit:</i>                  Exponents must be integers.   <i>GRAD Content Limit:</i>                  Items will be limited to linear and exponential.                  Expressions and equations must be able to be solved numerically. Table or graph required.                  Items will not require expressions or equations to be solved symbolically.</p>	1–2
<p><b>III.A.G6</b>                  Students will generate a table of values from a formula or equation.                  Students will graph the result of a formula or linear equation in ordered pair format on a grid.  <i>GRAD Content Limit:</i>  <math>x</math> and <math>y</math> axes may have different scales.                  Items do not require students to graph or generate a table of a non-linear relation.                  Formulas will only have unknowns to the first degree.                  Items may not require generating a linear equation from a table of values.                  Items may include real-world context (e.g., converting temperature).                  Given a continuous (i.e., individual points not indicated) linear graph, students will generate a table of values.                  Linear equations will be given in <math>y = mx + b</math> form.</p>	1–2
<p><b>III.A.G7</b>                  Students will translate a problem described verbally or by tables, diagrams or graphs, into suitable mathematical language, solve the problem mathematically and interpret the result in the original context.  <i>GRAD Content Limit:</i>                  Items may include real world context.</p>	1–2

<b>Strand III—Patterns, Functions and Algebra</b>		<b>Item Totals</b>
		<b>By Strand</b>
<b>Substrand B. Algebra (Algebraic Thinking)</b> Standard: Solve simple equations and inequalities numerically, graphically and symbolically. Use recursion to model and solve real-world and mathematical problems.		<i>continued</i>
		<b>By Substrand</b>
		7–12
<b>Benchmarks</b>		<b>By Benchmark</b>
<b>III.B.1</b> Students will translate among equivalent forms of expressions.  <i>Content Limit:</i> Items may include simplifying algebraic expressions involving nested pairs of parentheses and brackets; simplifying rational expressions; factoring a common monomial term from an expression; applying associative, commutative and distributive laws. A simplified expression should contain at most four terms with at most two variables per term.  <i>GRAD Content Limit:</i> Items may include simplifying algebraic expressions; simplifying rational expressions; factoring a common monomial term from an expression; applying associative, commutative and distributive laws. When applying distributive law, expressions may not contain 2 binomials. Expressions may not include nested pairs of parentheses or brackets. A simplified expression should contain at most two terms with at most one variable per term.		1–2
<b>III.B.3</b> Students will find equations of a line.  <i>Content Limit:</i> Items will provide two points on the line, a point and the slope of the line or the slope and y-intercept of the line. All answer options will be given in the same form within a MC item, either slope-intercept ( $y = mx + b$ ) or standard form ( $ax + by = c$ ). CR items must represent real-world contexts.  <i>GRAD Content Limit:</i> Items may require the student to generate the equation from the graph or identify the graph given the equation. Items will provide the slope and y-intercept of the line, when graph is not provided. Equations must be presented in slope-intercept form.		1–2

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<p><b>III.B.7</b> Students will solve linear equations and inequalities in one variable with numeric, graphic and symbolic methods. <i>Content Limit:</i> Forms of the linear equations or inequalities are not limited (e.g., <math>4(x + 5) - 3x = 6(x + 10)</math> is acceptable). Items may include context.  <i>GRAD Content Limit:</i> Items may include at most one application of the distributive property. Items will not include inequalities. Forms of the linear equations are limited to at most a binomial equaling a binomial. Items must have a numeric solution.</p>	1-2
<p><b>III.B.8</b> Students will determine solutions to quadratic equations in one variable with numeric, graphic and symbolic methods. <i>Content Limit:</i> All solutions are real. Solutions determined from a graph will be integer solutions. Items may include context.  <i>GRAD Content Limit:</i> All solutions are integers. Coefficient on second-degree term will always be 1.</p>	1-2
<p><b>III.B.11</b> Students will solve systems of two linear equations and inequalities with 2 variables using numeric, graphic and symbolic methods. <i>Content Limit:</i> Inequalities will only be solved graphically. Items may include context.  <i>GRAD Content Limit:</i> Items may include at most one application of the distributive property. Items will not include inequalities. Forms of linear equations are limited to at most a binomial equaling a binomial. Systems may be represented using graph, slope-intercept and table format. Systems are consistent and independent (i.e., solution is one ordered pair).</p>	1-2

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<p><b>III.B.12</b> Students will understand how slopes can be used to determine whether lines are parallel or perpendicular and determine equations for parallel lines and perpendicular lines.</p> <p><i>Content Limit:</i> Items may provide a line and a point not on that line. Items may require students to determine the equation of the line passing through a given point and parallel to a given line. Items may require students to determine the equation of the line passing through a given point and perpendicular to a given line. Items may include context.</p> <p><i>GRAD Content Limit:</i> Items may not require students to determine the equation of a line. Equations in problem and answer options must be in slope-intercept form.</p>	<p>1–2</p>
<p><b>III.B.G13</b> Students will use formulas with more than one variable to solve real-world and mathematical problems.</p> <p><i>GRAD Content Limit:</i> Formulas must be from a real-world context and may include powers (e.g., area, volume, <math>I=prt</math> or <math>d=rt</math>). Items may contain formulas with at most four variables. Roots are limited to square roots. Formula notation may not include subscripts. Formulas must be included within the item.</p>	<p>1–2</p>

<b>Strand IV—Data Analysis, Statistics and Probability</b>		<b>Item Totals</b>
		<b>By Strand</b>
		8–14
<b>Substrand A. Data and Statistics</b>		<b>By Substrand</b>
Standard: Represent data and use various measures associated with data to draw conclusions and identify trends. Understand the effects of display distortion and measurement error on the interpretation of data.		5–9
<b>Benchmarks</b>		<b>By Benchmark</b>
<b>IV.A.2</b> Students will use measures of central tendency and variability to describe, compare and draw conclusions about sets of data. <i>Content Limit:</i> Measures may be mean, median, maximum, minimum, range, standard deviation, quartile, percentile, mode or interquartile range (IQR).  <i>GRAD Content Limit:</i> Measures may be mean, median, maximum, minimum, range, quartile, mode or interquartile range (IQR). The interquartile range may be referred to conceptually, but the terminology “interquartile range” will not be used.	1–2	
<b>IV.A.3</b> Students will determine approximate line of best-fit and use the line to draw conclusions. <i>Content Limit:</i> Items will provide a scatter plot (coordinates of points on scatter plot are integers) or data set.  <i>GRAD Content Limit:</i> Equations are limited to linear equations only. Items will provide a scatter plot (coordinates of points on scatter plot are integers). Line of best fit may be provided and asked to draw conclusions.	1–2	

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<p><b>IV.A.G8</b> Students will analyze histograms, bar graphs, circle graphs, stem-and-leaf plots and box-and-whisker plots. <i>GRAD Content Limit:</i> Graphics may have at most ten data categories. Circle graphs may have at most eight sectors. Scales are in increments appropriate to the application. Histogram intervals must be consistent. Items may involve:</p> <ul style="list-style-type: none"><li>• Reading and interpreting</li><li>• Identifying trends and patterns and make predictions</li><li>• Solve problems using information presented in the graph</li></ul>	<p>2–3</p>
<p><b>IV.A.G9</b> Students will understand the meaning of and be able to compute minimum, maximum, range, median, mean and mode of a data set. <i>GRAD Content Limit:</i> At most twenty numbers in the data set. Numbers used are less than 300. Items may ask which values (mean, median, mode, range) “best” describes a data set in context and identify justification. Items may require the calculation of quartiles. The interquartile range may be referred to conceptually, but the terminology “interquartile range” will not be used.</p>	<p>1–2</p>

<b>Strand IV—Data Analysis, Statistics and Probability</b>		<b>Item Totals</b>
		<b>By Strand</b>
		<i>continued</i>
<b>Substrand B. Probability</b>	Standard: Use appropriate counting procedures, calculate probabilities in various ways and apply theoretical probability concepts to solve real-world and mathematical problems.	<b>By Substrand</b>
		3–5
<b>Benchmarks</b>		<b>By Benchmark</b>
<b>IV.B.1</b>	<p>Students will select and apply appropriate counting procedures to solve real-world and mathematical problems.</p> <p><i>Content Limit:</i>                      Items may involve computing probabilities.                      Items may include combinations and permutations.</p> <p><i>GRAD Content Limit:</i>                      Items may involve determining sample space and/or computing probabilities.                      Items may not include formulas.                      Solutions may have at most 24 possibilities.</p>	1–2
<b>IV.B.2</b>	<p>Students will calculate probabilities and relate the results in real-world and mathematical problems.</p> <p><i>Content Limit:</i>                      Items may use area, trees, unions and intersections to calculate probabilities.                      Items may involve both the concept of mutually exclusive events or not mutually exclusive events.                      Items may involve independent or dependent events.                      Items may involve conditional probability.</p> <p><i>GRAD Content Limit:</i>                      Items may involve independent events.                      Items will not involve conditional probability.</p>	1–2

<p><b>IV.B.3</b> Students will use probability models in real-world and mathematical problems. <i>Content Limit:</i> Models may include area and binomial models. Binomial probabilities will involve at most 4 events. <i>GRAD Content Limit:</i> Binomial probabilities will involve at most 3 events.</p>	<p>1-2</p>
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<b>Strand V—Spatial Sense, Geometry and Measurement</b>		<b>Item Totals</b>
		<b>By Strand</b>
		6–11
<b>Substrand A. Spatial Sense</b>		<b>By Substrand</b>
Standard: Use models to represent and understand two- and three-dimensional shapes and how various motions affect them. Recognize the relationship between different representations of the same shape.		1–2
<b>Benchmarks</b>		<b>By Benchmark</b>
<p><b>V.A.1</b> Students will use models and visualization to understand and represent various three-dimensional objects and their cross sections from different perspectives.</p> <p><i>Content Limit:</i> Items are limited to top view, side view, front view or net. Shapes are limited to polyhedra, combinations of polyhedra, cylinders and cones. No figures will be oblique. All visible sides of views are clearly labeled. Prisms will have a base with at most six sides. Pyramids will have a base with at most six sides. Cross sections are limited to rectangular prisms, cones, cylinders, rectangular pyramids and triangular pyramids.</p> <p><i>GRAD Content Limit:</i> Shapes are limited to prisms, pyramids, cylinders and cones. Prisms will have a base with at most four sides. Pyramids will have a base with at most four sides.</p>		1–2

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<b>Strand V—Spatial Sense, Geometry and Measurement</b>		<b>Item Totals</b>
		<b>By Strand</b>
		<i>continued</i>
<b>Substrand B. Geometry</b>		<b>By Substrand</b>
Standard: Apply basic theorems of plane geometry, right triangle trigonometry, coordinate geometry and a variety of visualization tools to solve real-world and mathematical problems.		4–7
<b>Benchmarks</b>		<b>By Benchmark</b>
<p><b>V.B.1</b> Students will know and use theorems about triangles and parallel lines in elementary geometry to justify facts about various geometrical figures and solve real-world and mathematical problems.</p> <p><i>Content Limit:</i> Theorems may include criteria for two triangles to be congruent or similar. Theorems may include facts about angles formed by parallel lines cut by a transversal. Items may involve the application of these theorems to solve real-world and mathematical problems involving other plane figures.</p> <p><i>GRAD Content Limit:</i> Items will require knowledge and use of theorems. Items will not require students to use theorems for justification. Items must include context or diagram.</p>	1–2	
<p><b>V.B.3</b> Students will use properties of two- and three-dimensional figures to solve real-world and mathematical problems.</p> <p><i>Content Limit:</i> Use 3.14 as an approximation for <math>\pi</math>. Situations may include finding area, perimeter, volume and surface area. Situations may include applying direct or indirect methods of measurement. Situations may include applying the Pythagorean theorem and its converse. Situations may include properties of 45-45-90 and 30-60-90 triangles.</p> <p><i>GRAD Content Limit:</i> Situations may include applying the Pythagorean theorem but not its converse. Situations will not include properties of 45-45-90 and 30-60-90 triangles. Limits on shapes in V.C.G.2 apply.</p>	2–3	

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**V.B.4**

Students will apply the basic concepts of right triangle trigonometry to determine unknown sides or unknown angles when solving real-world and mathematical problems.

*Content Limit:*

Concepts may include sine, cosine and tangent.

Items will not require the use of the reciprocals or inverses of sine, cosine and tangent.

Items will provide a table of decimal approximations of three trigonometry values for each angle given in the item or students may use trigonometry values from a calculator.

*GRAD Content Limit:*

Items must include diagram.

**Mathematics GRAD**  
**V.B.1, V.B.3**

1–2

<b>Strand V—Spatial Sense, Geometry and Measurement</b>		<b>Item Totals</b>
		<b>By Strand</b>
		<i>continued</i>
<b>Substrand C. Measurement</b>		<b>By Substrand</b>
Standard: Use the interconnectedness of geometry, algebra and measurement to explore real-world and mathematical problems.		1–2
<b>Benchmarks</b>		<b>By Benchmark</b>
<p><b>V.C.G1</b>                      Students will make calculations involving time, length, area, volume, weight and mass choosing appropriate units to calculate, measure and record.  <i>GRAD Content Limit:</i>                      Appropriate U.S. customary units are inches, feet, yards, miles, fluid ounces, cups, pints, quarts, gallons, ounces and pounds.                      Metric prefixes may include milli, centi, and kilo.                      Items may not combine systems within one item.                      Time units are years, months, weeks, days, hours, minutes and seconds.                      Time problems may include reading arrival/departure schedules.                      Items may include measurement conversions.</p>		0–1

*Mathematics GRAD*

**V.C.G2**

Students will use formulas to solve real world and mathematical problems.

*GRAD Content Limit:*

Items may include determining the surface area or volume of shapes.

- Shapes are limited to cubes, prisms and cylinders.
- Pictures or diagrams may be used but are not required.
- The radius or diameter is supplied for cylinders.
- Answer options may be left in terms of  $\pi$  (e.g.,  $7\pi$ ).
- Non-rectangular prisms must provide the area of the base.

Items may include calculating perimeter and area of two-dimensional figures obtained by putting together triangles, parallelograms and sectors of circles to solve real-world and mathematical problems.

- Items must provide a picture or diagram.

Items may include calculating the radius, diameter, circumference and area of a circle.

- Given the diameter or radius, items may require students to determine area or circumference.
- Given the circumference, items may require students to determine radius, diameter or area.
- Radii must be greater than 2.

Grade 11 Formula Sheet will be provided. (See copy of grade 11 formula sheet in Grade 11 Item Sampler).

0–1